

Leverage and Path Dependence for Perpetual Swaps

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This article explores how leveraged perpetual swap positions exhibit path dependence, meaning the sequence of price movements determines which positions survive liquidation. The analysis demonstrates that leveraged perps behave similarly to synthetic options and introduces the concept of "Expected Time to Liquidation" to help traders understand the implicit duration of their leveraged positions.

1 Understanding Path Dependence in Leveraged Perps

Perpetual swaps are typically considered delta-one instruments with 1-to-1 payoffs matching underlying asset prices. However, leverage introduces complexity: "the sequence of spot price affects which positions survive and which positions get liquidated."

The article presents simulations showing liquidated positions (in red) versus surviving positions (in blue) for a 10x leveraged perpetual swap with 50% volatility.

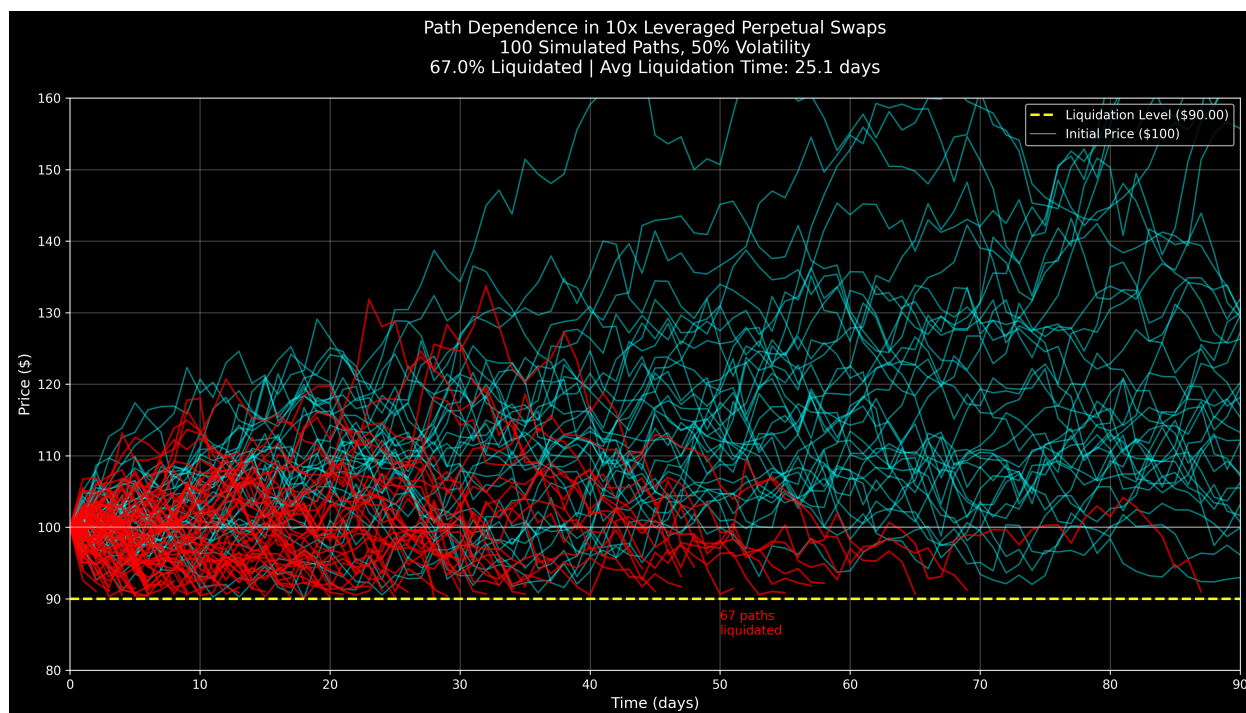


Figure 1: Perpetual swap liquidation paths

When graphed, leveraged long perpetual payoffs resemble call option payoffs when margin isn't topped up and liquidation occurs.

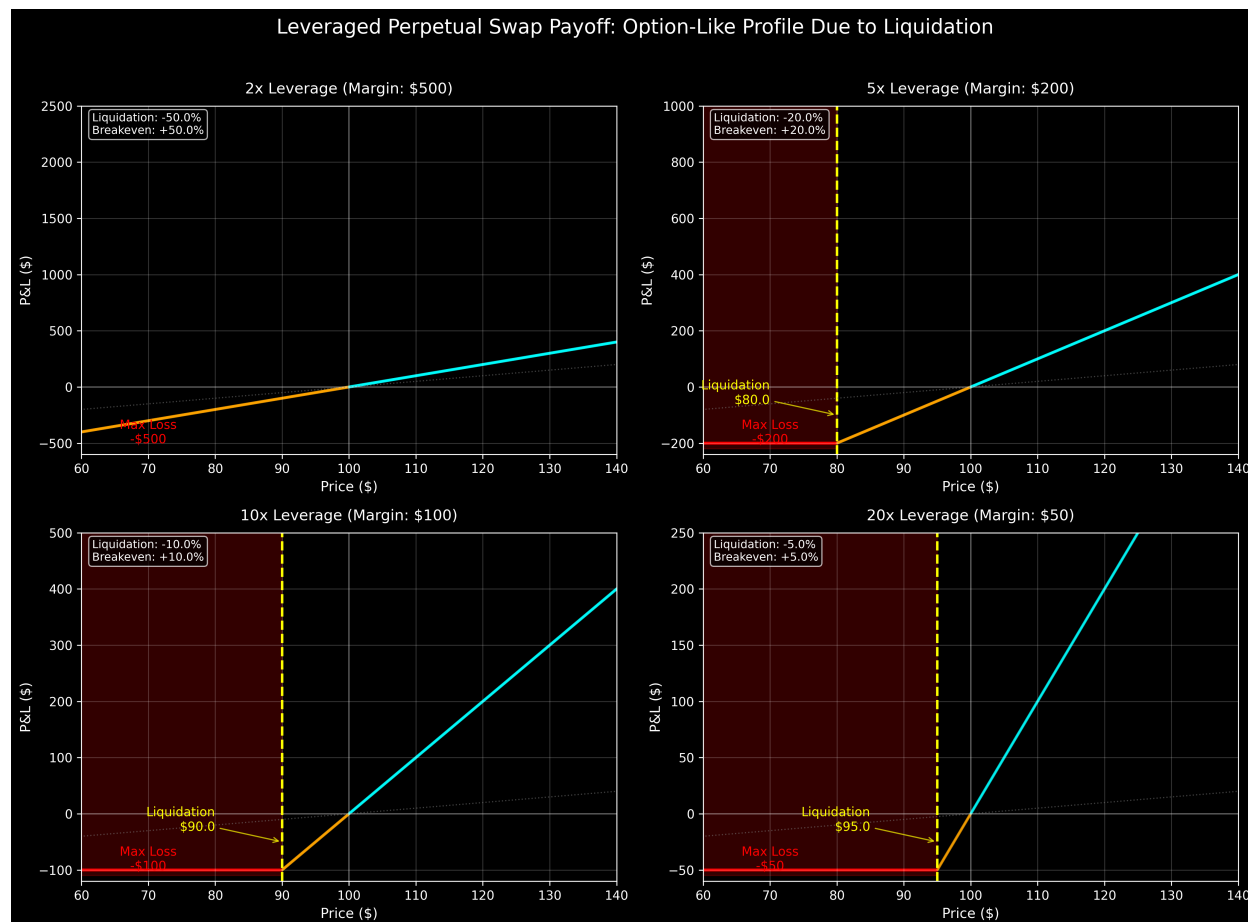


Figure 2: Leveraged perpetual payoff

2 Expected Time to Liquidation Framework

Rather than expecting liquidation, traders benefit from understanding "how long a perp position is expected to survive can provide helpful intuition on the implicit duration of the position expressed."

Key Variables:

- S_0 : Initial entry price
- L : Leverage ratio
- σ : Annualized volatility
- μ : Drift rate (market trend minus funding rate)
- B : Liquidation barrier = $S_0(1 - 1/L)$
- τ : First passage time until liquidation

Price Dynamics Model: Price follows geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

Where W_t represents standard Brownian motion.

Converting to log-space with $X_t = \ln(S_t/S_0)$:

$$dX_t = (\mu - \sigma^2/2)dt + \sigma dW_t \quad (2)$$

The liquidation barrier in log-space: $b = \ln(1 - 1/L)$

Expected Time to Liquidation Formula:

$$E[\tau] = \frac{-\ln(1 - 1/L)}{\mu + 0.5\sigma^2} \quad (3)$$

For days:

$$E[\tau_{\text{days}}] = 365 \times \frac{-\ln(1 - 1/L)}{\mu + 0.5\sigma^2} \quad (4)$$

3 Practical Implications

The heatmap analysis reveals that a 50x leveraged long position in a 150% volatility asset faces liquidation within approximately 6.6 days under zero drift and funding conditions.

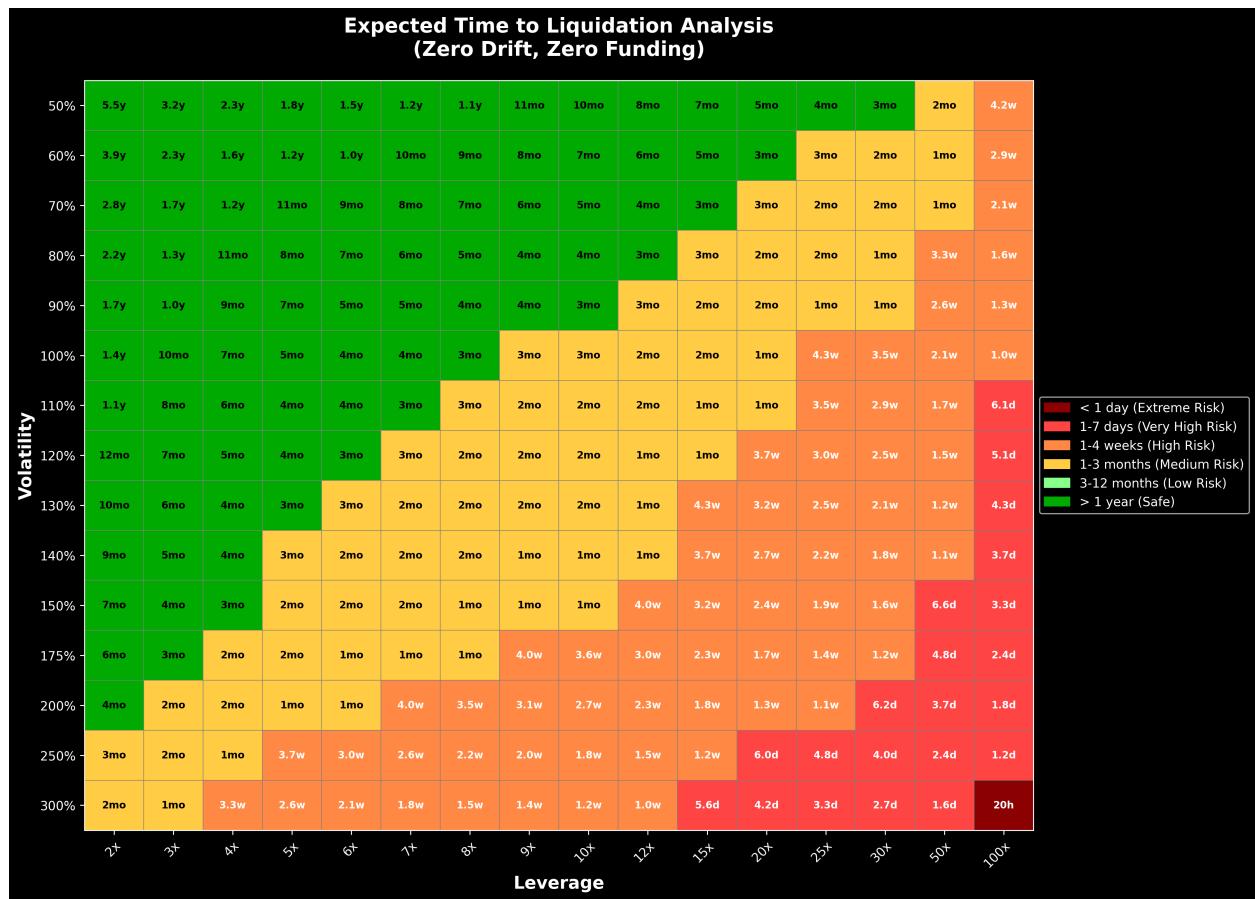


Figure 3: Liquidation heatmap

Key insights include:

- Higher leverage reduces expected time to liquidation disproportionately
- Funding costs further compress position duration
- Lower funding rates (via products like Ethena or HLP) enable longer-term leveraged positions
- Traders should calibrate leverage based on expected thesis timeframe and asset volatility

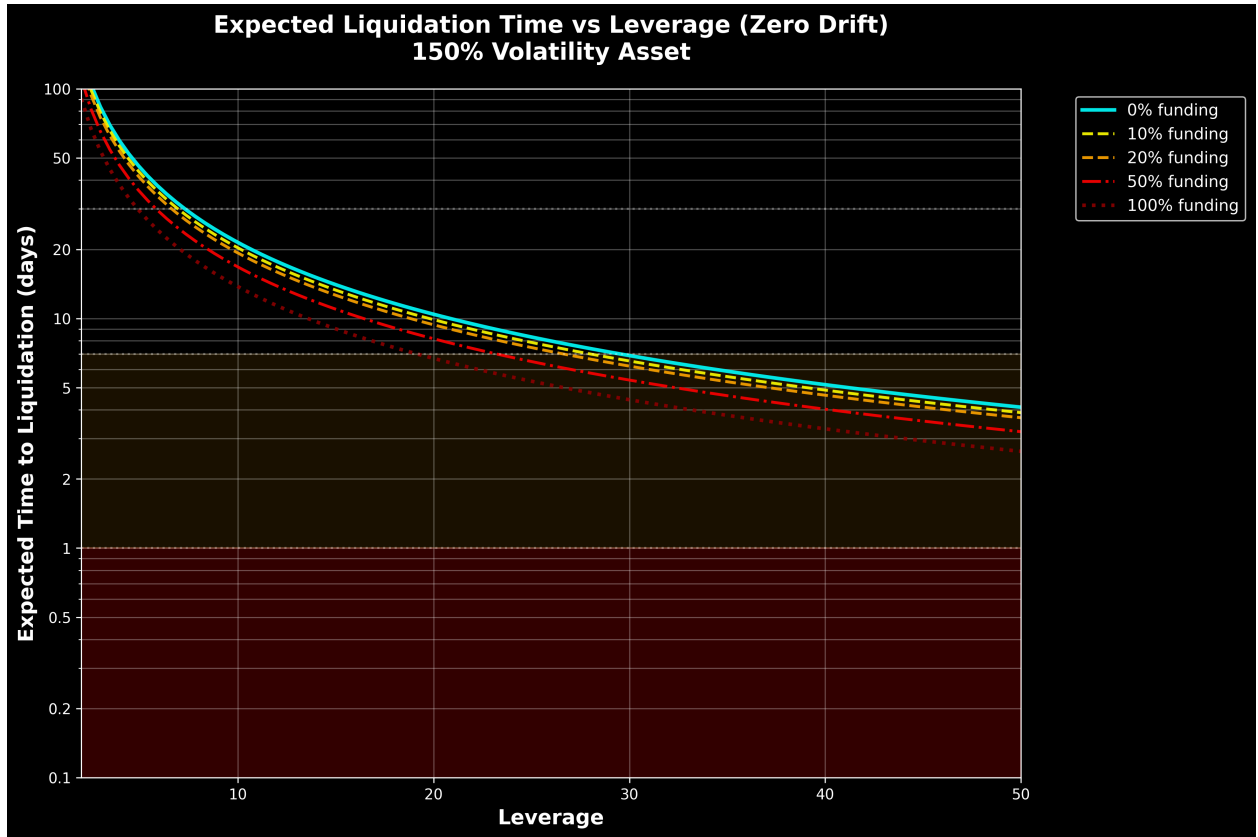


Figure 4: Median liquidation time by leverage and funding

The analysis underscores why perpetual exchange liquidity and funding rate mechanisms significantly impact trader behavior and position sustainability.

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